Working Paper
Series

FINRISK
National Centre of Competence in Research
Financial Valuation and Risk Management

National Centre of Competence in Research
Financial Valuation and Risk Management

Working Paper No. 216

Economic Benefit of Powerful Credit Scoring

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First version: September 2004
Current version: April 2005

This research has been carried out within the NCCR FINRISK project on "Conceptual Issues in Financial Risk Management".

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The National Centres of Competence in Research (NCCR) are a research instrument of the Swiss National Science Foundation.

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Schweizerischer Nationalfonds
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Economic Benefit of Powerful Credit Scoring *

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First Version: November, 2004

This Version: February 13, 2005

*The content of this paper reflects the personal view of the authors, in particular, it does not represent the opinion of Credit Suisse. The authors thank the seminar participants at the University of Zürich. Markus Leippold acknowledges the financial support of the Swiss National Science Foundation (NCCR FINRISK).

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Economic Benefit of Powerful Credit Scoring

ABSTRACT

In this paper, we study the economic benefits from using credit scoring models. We contribute to the literature by relating the discriminatory power of a credit scoring model to the optimal credit decision. Given the Receiver Operating Characteristic (ROC) curve of the credit scoring model, we derive *a)* the profit-maximizing cutoff regime and *b)* the pricing curve. In addition, we study a stylized loan market model with banks that differ in the quality of their credit scoring model. We find that profitability varies substantially among lenders. More powerful credit scoring models lead to economically significant differences in credit portfolio performance.

JEL Classification Codes: D40, G21, H81

Key Words: Bank loan pricing; Credit scoring; Discriminatory power; Receiver Operating Characteristic (ROC).

In this paper, we investigate the economic benefit of credit scoring models. Ordinal performance measures such as, e.g., the Receiver Operating Characteristic (ROC) curve, are widely used to assess the discriminatory power of credit scoring and rating models. However, performance statistics and common lending practice seem to be two separate worlds. We show how to reconcile ordinal power measures with metrics like profit and loss. In addition, we present a simple loan market model where banks with different credit scoring models compete for loans. By calibrating the model, we find that higher discriminatory power translates into significant profit improvement.

For banking institutions, loans are often the primary source of credit risk. Traditional lending practice has been to grant loans that have a positive net present value (NPV) and to deny those that do not. Recently, the use of statistical models has increased significantly. To assess the risk of these loans, banks use credit scoring models and credit ratings to estimate default risk on a single obligor basis.

Loans to small and medium sized companies, mostly unrated firms, are an important portion of most banking institutions' portfolios. Since the individual amount of exposure to such firms is often relatively small, it is uneconomical to devote extensive resources to the credit analysis. Therefore, for such borrowers, banks use credit scoring models instead of rating models. The credit scoring model should optimize both the likelihood of a bad obligor being accepted and the likelihood of a good obligor being rejected. Similarly, in the case of a pricing-based lending, a credit scoring model with low discriminatory power can lead to underpricing of bad and overpricing of good loans. For a recent survey on the use of credit scoring models, we refer to Thomas (2000) and Thomas, Edelmann, and Crook (2002).

In evaluating the performance of credit scoring models, it is common practice to use ordinal measures such as, e.g., the Receiver Operating Characteristic (ROC) curve and its associated discriminatory power statistics. However, it is not a priori clear how discriminatory power is linked with credit decision making and credit risk pricing. Establishing such a link is essential for the profitability of the bank's credit business. If in a market with several suppliers of loans in which, by means of a higher default prediction accuracy, one bank knows better than its competitors about the quality of loans, the information advantage may translate into better profitability figures.

In this paper, we show how lenders can incorporate the scoring model and its ROC-based performance measure into traditional lending practices, based on NPV considerations. By relating the discriminatory power of a credit scoring model to the optimal credit decision, we derive *a)* the profit-maximizing cutoff regime and *b)* the pricing curve. In addition, to analyze the economic impact of discriminatory power, we study a stylized loan market with banks that differ in the quality of their credit scoring model. We find that profitability varies substantially among lenders. More powerful credit scoring models lead to economically significant differences in credit portfolio performance.

The paper is organized as follows. Section I explains the Receiver Operating Characteristic (ROC) concept to assess the discriminatory power of credit scoring models. Section II describes how profit-optimal credit decisions can be deduced from ROC statistics. In section III, we present a stylized loan market model under different market regimes. For each regime, we calculate different economic figures, like market share and profit. Section IV concludes.

I. Discriminatory power

Credit scoring models can err in two ways. First, the model may indicate low risk when, in fact, the risk is high. This error, typically referred to as α-error, corresponds to the assignment of high credit quality to obligors who nevertheless default or come close to defaulting. The cost of the bank is the loss of credit amount and/or interest. Secondly, the model may indicate high risk when, in fact, the risk is low. This error, usually referred to as β-error, relates to low-rated firms that should, in fact, be rated higher. Potential losses resulting from this second type of error include the loss of return and fees as well as a drop in market share when loans are either turned down or lost through non-competitive pricing. Table 1 gives an overview of the various costs occurring from α- and β-errors.

[Table 1 here]

There exist several methods to measure the statistical performance of credit scoring models. One of the most applied methods is the Receiver Operating Characteristic (ROC). The ROC analysis is a technique originally used in medicine, engineering, and psychology to assess the performance of diagnostic systems and signal recovery techniques (see, e.g., Egan (1975)).

The ROC curve is a two-dimensional measure of classification performance and visualizes the information from the Kolmogorov-Smirnov statistics. It is constructed by calculating the α- and β-errors for every possible cutoff level t. The two sets of errors correspond to the coordinates of the Receiver Operating Characteristic (ROC) curve. (See also Sobehart and Keenan (2001) as well as Engelmann, Hayden, and Tasche (2003) for a discussion of measuring discriminatory power of credit scoring models).

The ordinate of the ROC curve is scaled as the hit rate, i.e., one minus the α-error, under the null hypothesis that high scores translate into high default probabilities

$$\begin{aligned} 1-\alpha(t) &= \mathbb{P}\left\{S > t | Y = 1\right\} \\ &=: \mathbb{P}\left\{S_D > t\right\}, \end{aligned}$$

where S is the credit score and S_D is the conditional credit score of defaulters. The abscissa is scaled as the false alarm rate (β-error)

$$\begin{aligned} \beta(t) &= P\left\{S > t | Y = 0\right\} \\ &=: P\left\{S_{ND} > t\right\}, \end{aligned}$$

where S_{ND} is the conditional credit score of non-defaulters. The construction of the ROC curve is illustrated in Figure 1, where we show possible distributions of rating scores for defaulting and non-defaulting obligors. For a perfect rating model, the left distribution and the right distribution would be separate. For a real credit portfolio, perfect discrimination is not possible. Both distributions will overlap. In Figure 1, the dark area under the population of defaulters represents the α-error. The shaded area under the population of non-defaulters represents the proportion of false alarms (β-error) generated by the model in response to the particular cutoff score t.

[Figure 1 here]

Figure 2 plots the ROC curves for four different models: Model I, Model II, as well as a random and a perfect rating model. In Figure 2, the diagonal line corresponds to random forecasts. When the curve bows away from the diagonal line to the upper left corner, this indicates an improvement of the model's performance. Thus, the ROC of a powerful rating model is steep at the left end and flat near the point $(0,1)$. Similarly, the larger the area below the ROC curve, the better the model. This area is usually called the AUROC. A nice interpretation of the AUROC is given by, e.g., Bamber (1975) and Hanley and McNeil (1982).

[Figure 2 here]

Formally, under the null hypothesis that high score values indicate low creditworthiness, AUROC is defined as

$$\text{AUROC} \;=\; -\int_{-\infty}^{\infty} \underbrace{\mathbb{P}\left\{S_D > y\right\} d\mathbb{P}\left\{S_{ND} > y\right\}}_{\text{area rectangle}} - \int_{-\infty}^{\infty} \underbrace{\frac{1}{2}\mathbb{P}\left\{S_D = y\right\} d\mathbb{P}\left\{S_{ND} > y\right\}}_{\text{area triangle}}.$$

By transforming we get

$$\begin{aligned}
\text{AUROC} &= \int_{-\infty}^{\infty} \left[\mathbb{P}\left\{S_D > y\right\} + \frac{1}{2}\mathbb{P}\left\{S_D = y\right\}\right] dF_{S_{ND}}(y) \\
&= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left[\mathbf{1}_{\{x>y\}} + \frac{1}{2}\mathbf{1}_{\{x=y\}}\right] dF_{S_D}(x)\, dF_{S_{ND}}(y) \\
&= \mathbb{P}\left\{S_D > S_{ND}\right\} + \frac{1}{2}\mathbb{P}\left\{S_D = S_{ND}\right\},
\end{aligned}$$

where $F_{S_{ND}}$ and F_{S_D} are the non-defaulters' and defaulters' distribution functions, respectively. The last line follows by the assumption of independent draws out of the two populations. For continuous random variables, we have $\mathbb{P}\left\{S_D = S_{ND}\right\} = 0$. An AUROC of 0.5 (area of the orthogonal, isosceles triangle) reflects random forecasts, while AUROC $= 1$ (area of the square) implies perfect forecasts. For any reasonable rating model, the AUROC lies between 0.5 and 1.

We note that the discussion of power must always take place with the understanding that, if the goal is to compare two models, calculation of power needs to be done on the same population. Power statistics are especially sensitive to the sample chosen when the number of defaults is limited, as is typically the case in commercial lending. Therefore, differences in samples may lead to different assessments of power (see also Hamerle, Rauhmeier, and Roesch (2003) and Stein (2002)). In the subsequent analysis, we report the discriminatory power as a measure for the entire loan market and, consequently, on the same population.

II. Credit Decision

In this section, we extend the ROC analysis and link it to an NPV analysis to deduce optimal credit decisions. First, we discuss the derivation of profit-maximizing threshold values (cutoff regime). Second, we show how the pricing curve can be derived from the ROC curve (pricing regime).

A. Cutoff regime

The basic use of ROC analysis is to provide guidance for setting the lending cutoffs. The user can define model scores below which a loan will be granted and above which it will not. However, the determination of the cutoff level is an arbitrary choice, most often based on qualitative arguments such as, e.g., business constraints. From a value-maximizing perspective, such a determination is in general suboptimal.

A more rigorous criterion can be derived with knowledge of the prior probability of default and the associated costs and revenues. To this end, we make the simplifying assumptions that there are exogenously determined costs of default and a single market premium R for bearing credit risk.[1] Given this risk premium, a bank can either accept or reject a loan. Table 2 gives an overview of the cash-flows involved in case of default and in case of non-default, respectively.

[Table 2 here]

Given the default probability and the expected cash-flows, net present value NPV per unit of credit amount (i.e., one US Dollar) dependent on the credit score t can be written as

$$\mathrm{NPV}(t) = -1 + \frac{1}{1+\delta}\left[\mathbb{P}\{Y=1|S=t\}(1-LGD) + \mathbb{P}\{Y=0|S=t\}(1+R+C)\right], \quad (1)$$

where δ is the risk-adjusted discount rate. We assume all quantities to be adjusted for a one-period time horizon. LGD denotes loss-given default and includes recovery costs. Recovery costs or workout fees depend heavily on law enforcement and liquidity of the collateral. We treat LGD as an exogenously determined constant value or expected value, respectively.[2] In equation (1), R represents the interest to be paid at maturity, $\mathbb{P}\{Y=1|S=t\}$ is the conditional probability of default given knowledge of the credit score. Relationship managers often use the argument of "strategic value" when taking on seemingly negative-NPV loans. We capture this strategic value by C, which is an (real) option to make follow-on business such as, e.g., private banking activities.

In the subsequent analysis, we will assume that the bank does not invest in negative NPV-projects. Thus, the lender rejects all obligors that do not fulfill either of the following two inequalities (2) and (3) below:

$$R \geq \frac{\mathbb{P}\{Y=1|S=t\}}{\mathbb{P}\{Y=0|S=t\}} LGD - C + \frac{\delta}{\mathbb{P}\{Y=0|S=t\}} \quad (2)$$

$$\geq \frac{\mathbb{P}\{Y=1|S=t\}}{\mathbb{P}\{Y=0|S=t\}} LGD - C \quad (3)$$

$$= -\frac{\mathbb{P}\{Y=1\}\frac{d\alpha(t)}{dt}}{\mathbb{P}\{Y=0\}\frac{d\beta(t)}{dt}} LGD - C \quad (4)$$

$$= -\frac{\mathbb{P}\{Y=1\}}{\mathbb{P}\{Y=0\}}\frac{d\alpha(\beta(t))}{d\beta(t)} LGD - C. \quad (5)$$

A bank loan that fulfills inequality (3) is profitable. If it meets inequality (2), then the loan deal adds value to the bank's credit portfolio. We deduce equation (4) from the fact that

$$\begin{aligned}\alpha(t) &= \mathbb{P}\{S \leq t|Y=1\} \\ &= \frac{1}{\mathbb{P}\{Y=1\}}\int_{-\infty}^{t}\mathbb{P}\{Y=1|S=s\}\,dF(s), \\ 1-\beta(t) &= \mathbb{P}\{S \leq t|Y=0\} \\ &= \frac{1}{\mathbb{P}\{Y=0\}}\int_{-\infty}^{t}\mathbb{P}\{Y=0|S=s\}\,dF(s),\end{aligned}$$

where F is the distribution function of random variable S. We use the differential quotient somewhat informally in equation (4). In practice, the curve defined by a ROC analysis may be a step function and is therefore not differentiable. Nevertheless, we use it to convey the conventional meaning when interpreting ROC curves as if they were continuous and differentiable at least once.

Therefore, the required risk premium $R(t)$ for a specific credit risk score t increases *a)* with the steepness of the ROC curve, i.e., with $-\frac{d\alpha(\beta)}{d\beta}$, *b)* with the discount factor δ, *c)* with the loss-given default LGD, and *d)* with the expected default frequency $\mathbb{P}\{Y=1\}$. In contrast, the required risk premium $R(t)$ decreases with the value of the real option C.

In equation (3), we can set the discount rate equal to zero, i.e., $\delta = 0$, the net present value equals profit. Rearranging equation (2), we arrive at

$$-\frac{d\alpha(\beta(t))}{d\beta(t)} \leq \frac{\mathbb{P}\{Y=0\}}{\mathbb{P}\{Y=1\}}\frac{R+C}{LGD} =: s. \tag{6}$$

The left-hand side of the inequality represents the slope of the ROC curve at point t. A bank certainly refuses all obligors with negative expected profit. Hence, the bank rejects all applicants with a score t and a corresponding slope of the ROC curve higher than s. The numerator in (6) represents the probability-weighted opportunity cost of withholding lending to non-defaulters. The denominator represents the probability-weighted recovery cost of accepting defaulters.

We call the straight line in the ROC graph with slope s an iso-profit line. All points on the straight line achieve the same profit. The point at which the line with slope s forms a tangent to the ROC curve defines the optimal cutoff t^*. At this point, expected marginal profit is zero, i.e.,

$$-\mathbb{P}\{Y=0\}(R+C)\left.\frac{d\beta(t)}{dt}\right|_{t=t^*} = \mathbb{P}\{Y=1\}\,LGD\left.\frac{d\alpha(t)}{dt}\right|_{t=t^*},$$

or, using (4),

$$\underbrace{\mathbb{P}\{Y=0|S=t^*\}(R+C)}_{\text{conditional expected revenue}} = \underbrace{\mathbb{P}\{Y=1|S=t^*\}\,LGD}_{\text{conditional expected loss}} \tag{7}$$

The above findings are consistent with our general intuition. At the optimal cutoff, the probability-weighted marginal cost of a mistake has to equal the marginal benefit for a correct decision. In other words, all obligors with a conditional expected revenue higher than or equal to the conditional expected loss are accepted.

To give some intuition of how the optimal truncation values behave, consider the examples given in Figure 3 that shows how the cutoffs and the ranking of the various models change as we vary cash-flow assumptions. We plot four different cash-flow scenarios for each of the models, i.e., the perfect and the random model, and Model I and II. On the dashed line we have constant profit and the closer the line to the point $(0,1)$, the higher is the profit. For tangency between the ROC curve and iso-profit line, the slopes must be equal.

In practice, it is often the case that a particular model will outperform another model under some specific set of cash-flow assumptions, but can be disadvantageous under a different set of assumptions. If the ROC curve for two models cross, then neither model is unambiguously better than the other with respect to a general cutoff. When one ROC curve completely dominates the other, such as the perfect model, the dominant model will be preferred for any possible threshold value. For example, in the upper left panel of Figure 3, we are indifferent between Model I and Model II, but we prefer both of them over the random model. By slight changes of cash-flow assumptions, we prefer either Model I or Model II.

[Figure 3 here]

If we have to accept or reject applicants based on the random model, it is either optimal to accept all applicants

$$\mathbb{P}\{Y=1\}\,LGD < \mathbb{P}\{Y=0\}\,R,$$

or to refuse all

$$\mathbb{P}\{Y=1\}\,LGD > \mathbb{P}\{Y=0\}\,R.$$

Under some extreme cash-flow assumptions, it might be best to work with no model at all (see lower right panel of Figure 3).

Nowadays, risk spreads for bank loans are not stale and depend on the creditworthiness of each obligor. Banks set prices based on the predictions of their models, and there is no longer a single market-price for bank loans. Therefore, it is important to have a powerful model in order to derive competitive prices.

B. Pricing regime

In a pricing regime, the banks sets the prices of the loan according to the credit score. The bank will accept all applicants paying this price. Therefore, the main challenge for the bank lies in the determination of the appropriate and optimal price.

As an economically meaningful criteria, we assume that the bank does not invest in negative NPV projects. Then, the bank sets minimum prices for a borrower based on the predictions of its model, i.e.,

$$R(t) \quad \geq \quad -\frac{\mathbb{P}\{Y=1\}}{\mathbb{P}\{Y=0\}}\frac{d\alpha(\beta(t))}{d\beta(t)}LGD - C, \tag{8}$$

where $R(t)$ is the credit risk spread, now as a function of score value t. If equality applies the expected profit would be zero.

We rewrite (8) by introducing $k \geq 0$,

$$R(t) + C = \frac{\mathbb{P}\{Y=1|S=t\}}{\mathbb{P}\{Y=0|S=t\}}LGD + k. \tag{9}$$

In the long run, a bank cannot offer loans for a revenue $R(t) + C$ lower than the right hand side of (8). Therefore, this term marks a lower bound, i.e., a minimum price. Since the slope of the ROC curve determines the minimum price, we can link pricing rule and ROC curve, both graphically and functionally.

Figure 4 shows two credit scoring models with the same discriminatory power but different minimum pricing schemes. We also plot the random rating model. For this model, the slope of the ROC curve is one. Therefore, every borrower has to pay the same interest rate, ceteris paribus. The bank cannot apply a price discrimination strategy. On the other side, if a bank disposes of a perfect scoring model, all defaulting obligors are refused and the others have to pay the risk-free rate or even less if they are "relationship" customers. Note that with a steeper ROC curve, the bank can put in place a more effective price discrimination strategy.

[Figure 4 here]

C. Mixture of cutoff and pricing regime

Credit specialists question both cutoff and pricing regime, since both approaches do not reflect a real credit environment. A cutoff regime oversimplifies today's lending practice in which risk-adjusted pricing is common practice. A pure pricing regime has also its shortcomings. First, it is questionable whether the risk premium is strictly exogenous. One could imagine that a high risk premium may backfire, in the sense that it triggers a failure. A high risk premium may therefore increase the default probability. Secondly, one should challenge the assumption that obligors pick or change their current bank only based on slight pricing differences. A good relationship between an obligor and a bank is of value and might keep obligors from switching banks, even though they would have been charged a lower risk premium by another bank.

With the above arguments in mind, we suggest a different approach that consists of a mixture of cutoff and pricing regimes. We construct such a mixture model as follows. First, we start with the pricing rule (9). Unlike the pure pricing regime, the risk premium $R(t)$ is rounded, i.e., toward the next quarter of a percentage point. By doing so, we capture the impact of transactional costs. Slight changes in pricing do not necessarily lead to a loan deal for the bank that would offer at the lowest exact rate as in a pricing regime. If two or more banks offer at the same rounded rate, the bank is selected by random. Secondly, obligors whose risk premia exceed an upper threshold value are rejected. This assumption mitigates possible feedback effects of high risk premium on default events.

III. Loan Market Model

Given the profit-maximizing cutoffs or optimal pricing rules, a bank can run into severe adverse selection problems. A rational credit applicant closes the deal with the lender that provides the most favorable terms. When the bank derives the price from the credit score, a low-power model results in a non-competitive pricing system. Hence, a marginal power improvement may lead to a sizeable profitability increase.

Moreover, the quality of the credit scoring model may play a decisive role on how the market is shared among the competing banks. For example, imagine two financial institutions, one with a very high-power model (or even the perfect model), whereas the other bank does the credit business without any model. The first bank will experience almost no defaults, as most of them are absorbed by the second bank. Even worse, the second lender will place almost no loan contracts with non-defaulting obligors, due to overpricing.

To better understand the economic impact of scoring methods with given discriminatory power, we present a stylized loan market model and analyze the three market regimes as described in the previous section, i.e., the cutoff regime, the pricing regime, and the mixture of cutoff and pricing regime. The cutoff regime is rather conservative, since the bank uses the credit score to either accept or reject costumers at an exogenously given risk premium. The probability of acceptance depends on the credit score. In the pricing regime, the lender is able to either attract or alienate obligors by applying a “smart” pricing scheme. All applicants are offered loans with corresponding risk adjusted credit spreads. Hence, the accepting probability is fixed at one, but the credit risk premium varies with credit scores. By mixing cutoff and pricing, both probability of acceptance and credit spread are attached to the score. Therefore, the mixture regime is closer to common market practice.

We assume a market that is composed of three lenders with credit rating scores S_1, S_2, and S_3. By Y^*, we denote the unobservable creditworthiness. We make the following distributional assumption

$$\begin{pmatrix} S_1 \\ S_2 \\ S_3 \\ Y^* \end{pmatrix} \backsim N\left(\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & \rho & \rho & \rho_1 \\ \rho & 1 & \rho & \rho_2 \\ \rho & \rho & 1 & \rho_3 \\ \rho_1 & \rho_2 & \rho_3 & 1 \end{pmatrix}\right). \tag{10}$$

We note that, since S_1, S_2, S_3 and Y^* represent ordinal measures, we can assume normalized variables without loss of generality. By the principle of parsimony, we adopt equal correlation among credit scores ρ. If the correlation ρ is high and if there are differences in discriminatory power, then the result will be a high profit difference, ceteris paribus. The default indicator Y is defined as follows

$$Y = \begin{cases} 0 & : \quad Y^* \leq c, \\ 1 & : \quad Y^* > c, \end{cases}$$

where the threshold c is calibrated to match the unconditional probability of default $\mathbb{P}\{Y = 1\} = \Phi(-c)$. The conditional probabilities of default are computed as

$$\mathbb{P}\{Y = 1|S_i\} = \Phi\left(-\frac{c - \rho_i S_i}{\sqrt{1 - \rho_i^2}}\right), \tag{11}$$

$$\mathbb{P}\{Y = 1|S_i, S_j\} = \Phi\left(-\frac{c - \rho_i S_i - \frac{\rho_j - \rho\rho_i}{1-\rho^2}(S_j - \rho S_i)}{\sqrt{1 - \rho_i^2 - \frac{(\rho_j - \rho_i\rho)^2}{1-\rho^2}}}\right). \tag{12}$$

The conditional default probability (11) corresponds to the unconditional probability $\mathbb{P}\{Y = 1\}$, if credit score S_i and creditworthiness Y^* are independent, or equivalently if $\rho_i = 0$. Later, we will also investigate the economic impact when a bank finds out about the scoring method of a competitor and employs this knowledge to increase forecasting accuracy. Then, the conditional probability of default can be conditioned on two different scores. This conditional probability is computed in equation (12).[3]

A. Calibration and simulation

To calibrate the models, we resort to the experience of credit officers as well as statistical findings. Usually, we find high correlation between statistical credit scores, which is basically driven by a similar information set. Since the information for making default predictions is typically based on financial key figures, computed by balance sheet and income statement, the credit scores between two banks are highly correlated in absolute terms. Therefore, we fix ρ at 0.8. The remaining correlation coefficients ρ_1, ρ_2, and ρ_3 are set at 0.48, 0.5, and 0.52, that corresponds to AUROC levels in practice. An LGD of 0.4 results as a weighted average of collateralized and blank credits. We further fix the unconditional probability of default at $\mathbb{P}\{Y = 1\} = 0.02$ and the corresponding threshold at $c = 2.0537$. These values are in line with practical experience.

In the mixture regime, the prices are rounded toward the next quarter of a percentage point. Credit officers argue that they still keep obligors from moving, even if the bank's credit spread is, on average, around one eighth of a percentage point higher than risk premia of competitors, by virtue of connecting with customers. In case of a mixture regime, all credit applicants with rounded risk premia higher than 2.5% are rejected. In both the pricing and mixture regime, we set $k - C$ in (9) equal to 30 basis points for all obligors. In practice, customer relationship manager of typical retail banks rarely impose credit risk spreads higher than 2.5%. For the cutoff regime, we assume a constant risk premium R of 75 basis points. Finally, we fix the market potential for bank loans in the economy at USD 100 billion.

Starting from this basic setting, we are in a position to calculate different statistics for all three market regimes, e.g., AUROC, price per creditworthiness, market share per creditworthiness, market share defaulters and non-defaulters, loss, revenue, profit. To calculate these statistics, we use Monte-Carlo simulation. To briefly illustrate the simulation approach, we look at the first two draws from the Monte-Carlo simulation (see equation (10)),

$$\left\{ \begin{pmatrix} 0.4991 \\ -0.1177 \\ 0.0208 \\ 0 \end{pmatrix}, \begin{pmatrix} 1.5059 \\ 0.8904 \\ 1.6744 \\ 2.3263 \end{pmatrix}, \ \ldots \right\}. \tag{13}$$

The first draw represents a median client. One half of the population have better, the other half lower creditworthiness Y^*. The random credit scores of 0.4991 (Bank 1), -0.1177 (Bank 2), and 0.0208 (Bank 3), lead to default probabilities of 0.0193, 0.0074 and 0.0084 according to equation (11). The second draw represents a defaulting client, because the creditworthiness exceeds threshold $c = 2.0537$. The corresponding conditional probabilities are 0.0646, 0.0316, and 0.0830.

B. Cutoff regime

In case of evaluating the economic impact in a cutoff regime, the assumption is that all three banks lend at the same risk premium. Therefore, price is not a discriminator. In a first step, the obligor chooses, with equal probability, one bank at random. If the score of the borrower was below the cutoff of the credit scoring model the bank is using, the loan will be assigned to that bank. If the score was above the cutoff, the borrower picks, again at random, one of the remaining two banks. If the score was below that cutoff, the loan is assigned to the bank of second choice. If not, the borrower's score on the last model is compared to that model's cutoff. If not accepted at the last bank, it is assumed that the loan would be denied by all three banks.

For our calibration defined above, the optimal cutoffs t^*, according to (7), are 0.4628, 0.4912 and 0.5199. This means that Bank 3 is willing to accept more applicants than Bank 1 and Bank 2. Figure 5 depicts the derivation of profit-maximizing threshold values. By referring to sample vector one in (13) we see that this actual non-defaulter is rejected by Bank 1, credit score of 0.4991 is higher than cutoff of 0.4628, and accepted by Bank 2 and Bank 3. Therefore, the revenue of $R = 0.0075$ is expected to being split between Bank 2 and Bank 3. In the second simulation (defaulting loan) in (13), all three credit scores exceed the corresponding cutoffs. Thus, the potential borrower is rejected by all three lenders.

[Figure 5 here]

Figure 6 plots probability of acceptance versus creditworthiness, with y-axis labeling on the left, and plots expected offering premium versus creditworthiness, with y-axis labeling on the right. The risk premium is fixed at $R = 0.0075$ and not variable (green horizontal line). What we observe is that a median borrower has a probability of 0.7011 (0.7146, 0.7285) of being accepted by Bank 1 (Bank 2, Bank 3). Comparing the high-power model (Bank 3) to both the medium-power (Bank 2) and low-power model (Bank 1), the probability of acceptance is higher for "good" and lower for "bad" borrowers. The three curves intersect at around 91%. Hence, defaulting obligors, on the right hand side of the solid vertical line, have the greatest chance of getting a loan at Bank 1 (low-power).

[Figure 6 here]

Figure 7 plots market share versus creditworthiness, with y-axis labeling on the left, and plots expected revenue versus creditworthiness, with y-axis labeling on the right. A median loan exhibits a probability of 0.2760 (0.2857, 0.2960) of going to Bank 1's (Bank 2's, Bank 3's) loan portfolio. The expected revenues on a median loan are 20.7, 21.4 and 22.2 basis points, respectively, as can be observed from the green lines. The market share and the expected revenue curves both cross at around 91%.

[Figure 7 here]

Table 3 shows expected market share for the whole population, defaulters, and non-defaulters. Bank 3, applying the best-performing model, closes more loan deals and accepts less defaulting loans, compared with its competitors. Thus, a high-power model leads to a more profitable credit-portfolio and a smaller recovery portfolio. This discrepancy underscores the adverse selection problem for banks with weaker models in a competitive market.

[Table 3 here]

C. Pricing regime

Unlike cutoff, the pricing regime allows variable credit spreads. All banks derive risk premia for loans according to equation (9), with $k = 0.0030$, and with no relationship benefit, $C = 0$. This means, all three banks price each loan with conditional expected profit equal to 30 basis points, corresponding to their credit scoring models. "Conditional" refers to the information set which consists of the credit score. Therefore, using only the banks' credit scores, information about the baseline default rate and knowledge of the cash-flows of lending, all banks set prices for loans.

To illustrate the pricing and selection mechanism, we refer to the two sample vectors in (13). According to the pricing rule in equation (9), the two simulations result in exact premia of 109, 60, 64 basis points (first draw, non-defaulter) as well as 306, 161, 392 basis points (second draw, defaulter). In both simulation draws the loan is granted by Bank 2. In the first draw, Bank 2 makes a revenue of 60 basis points. However, in the second draw, Bank 2 has to write off a loss of 40%.

Figure 8 shows both probability of acceptance (fixed at one) and expected premium offered by the corresponding bank. Around two thirds of all the borrowers can expect lower risk premia at Bank 3 with the high-power model. The remaining third of the population consists of "bad obligors." They are attracted by the lower premium of Bank 1 and Bank 2 that are operating with the medium-power and low-power model, respectively.

[Figure 8 here]

Figure 9 plots the expected market share, left y-axis, and expected revenue, right y-axis, per creditworthiness. The better the credit quality of the obligor, the more likely the obligor is closing the deal with Bank 3 using the high-power model. Given a defaulting borrower, it is more probable that Bank 1, with a low-power model, is granting the credit. The probability that a defaulter (non-defaulter) closes the deal with the high-power Bank is 28.7% (39.6%). The corresponding probabilities for the low-power Bank are 38.2% and 27.5%, respectively. Aggregated market share numbers are shown in Table 4.

[Figure 8 and Table 4 here]

The above analysis shows that the bank pricing with the most powerful model, due to a more exact estimate of the probability of default, can price the loans more attractively for those borrowers that are less likely to default, and more expensively for those that are more likely to default. On average, the poorer credits end up going to banks with weaker models. Borrowers with low default probability will end up doing business with banks that apply more powerful scoring models. The bank using the weaker model is in effect creating adverse selection for itself. These differences in pricing and market shares will necessarily lead to different profits as displayed (see Table 4).

D. Mixture of pricing and cutoff regime

In the previous two subsections, we discussed the cutoff and pricing regimes. Credit officers and customer relationship managers alike would probably discard both regimes. Consequently, we propose a mixture model that is closer to an actual credit banking environment.

Consider again the sample vectors in (13). These draws result in exact premia of 109, 60, 64 basis points (first draw) as well as 306, 161, 392 basis points (second draw). Rounding these figures results in risk premia of 1%, 0.5%, 0.75% and 3%, 1.50%, 4%, respectively. Bank 2 gets both loans – the former (non-defaulter) for a revenue of 0.5%, the latter (defaulter) for a loss of 40%. Bank 1 and Bank 3 reject the defaulting obligor, since the risk premium would exceed the upper bound of 2.5%.

Figure 10 shows both the probability of acceptance and the expected risk premium. Unlike pricing and cutoff, both measures depend on the nonobservable creditworthiness. Bank 3 (high-power) is inclined to accept more creditworthy borrowers at lower risk premia than Bank 2 (medium-power) and Bank 1 (low-power). Right from the vertical line, where we find defaulting borrowers, the story goes the other way around. These findings on a microstructure can be confirmed on a market level, as illustrated in Figure 11.

[Figure 10 and 11 here]

Table 5 summarizes the findings in Figure 10 and 11. The profit difference among banks amounts to up to USD 69.1 millions. The mixture regime is more competitive than cutoff. In the cutoff regime, the profit difference reaches only USD 22.6 millions. However, the mixture regime is less aggressive than the pricing regime, in which we observe a maximum profit gap of USD 110.6 millions. Loosely speaking, profit differences in a cutoff and pricing approach determine lower and upper bounds for profit gaps in an actual banking environment.

[Table 5 here]

E. Impact of Model Improvement and Additional Information

The previous section restricts the analysis to a cross-sectional perspective. Here, we are concerned with the situation in which one of the competing banks changes its rating system from one period to the next. Within the mixture regime, we study two different settings. In the first setting, one competitor improves discriminatory power. In the second setting, the rating methodology of one lender becomes public, either on purpose or because information has leaked. Then, competitors will use the additional information in order to increase their forecasting accuracy.

E.1. Improving the Credit Scoring Model

In the first setting, Bank 1's summary statistic AUROC increases, from 0.8134 to 0.8300, i.e., by means of switching from an expert-based to a statistical scoring procedure. By comparing Figure 10 to Figure 12, we observe a steeper pricing curve. True non-defaulters have to pay less, actual defaulters more than before. For a median borrower, the expected offered risk spread shifts from 83 basis points to 79 basis points. Similarly, Bank 1 would have accepted less investment grade and more loans close to default than its competitors. After the switch to the more accurate model, Bank 1 performs better but still lags Bank 3.

Bank 1's rating switch affects competitors as well, since market shares change. A median borrower now features a probability of 0.372 of heading towards Bank 1, up from 0.286, as depicted by Figure 13. Given this median borrower, Bank 1 (Bank 2, Bank 3) will now make higher (lower) expected revenue of 20.2 (19.3, 21.2) basis points, compared to 18.8 (20.6, 22.6) basis points before. Summarizing in Table 6, we can double-check that profitability and market share figures have changed in favor of Bank 1 and at the expense of both Bank 2 and Bank 3.

E.2. Additional Information

We now assume that Bank 1 is applying its improved scoring model, but somehow the rating knowledge has leaked. Bank 1 can only resort to its own scoring model, but Bank 2 and Bank 3 combine their own model with Bank 1's model for their default estimates. Clearly, the combined models achieve higher accuracies than their stand-alone counterparts.

Figure 14 and Figure 15 plot expectations when Bank 2 and Bank 3 are able to use Bank 1's rating model in order to improve their default forecasts. We refer to Table 7 for market statistics. Not surprisingly, Bank 1's profit drops. Its profit difference amounts to USD 11.3 million, compared to the situation when the rating methodology is not public. Hence, the information leak offsets a large part of a profit increase caused by an improved scoring methodology. On the other side, both Bank 2 and Bank 3 can improve their profits significantly by around USD 60 million, just by knowing the rating methodology of one competitor.

IV. Conclusion

In this paper, we provide rules how the credit score can be used for loan pricing by linking the ordinal ROC curve with the pricing curve. We show that the slope of the ROC curve enters the pricing rule. It turns out that the credit spread increases with the steepness of the ROC curve. When the loan market does not allow risk adjusted pricing, we derive the profit-optimal cutoff level values from the ROC statistics.

The profitability of a rating model depends also on other competitors' discriminatory power as well. In a stylized loan market, we study the economic impact of the discriminatory power of the scoring models. Weak models will attract more "bad" borrowers and therefore, the competitor with a low discriminatory power will incur lower revenues and bigger losses. In case of a cutoff regime, banks judging obligors based on a poor, unsound credit rating system grant more loans to subsequent defaulters and refuse more non-defaulting borrowers. We show, in quantitative terms, that these banks become unwittingly market leader in the segment of distressed loans, resulting in a sizable recovery portfolio. In case of pricing and mixture regime, a bank with a low-power model attracts bad customers through (too) high prices for credit-worthy and (too) low risk premia for not-credit-worthy borrowers. An increase in the discriminatory power of one bank affects the profits as well as the market shares of all other lenders.

Common to all regimes is the fact that the better the scoring model, the lower is the risk of adverse selection and the higher the added value to the bank. A lender can significantly increase its loan portfolio by improving its rating system, with the positive side effect that the recovery portfolio decreases. The increase takes place even in a saturated loan market. Therefore, in an emerging and growing loan market, the rise in profit and market share will be even greater.

Credit scoring is regarded as a core competence of commercial banking. We end our discussion of the loan market model by giving a quantitative example that highlights the economic disadvantage when this core competence is lost. If competitors can exploit another bank's rating knowledge, they can improve their own profits at the expense of the bank whose knowledge has leaked. Therefore, banks should pay attention to whom they communicate their scoring methodologies.

Appendix

We construct a credit market with n banks as supplier of loans and obligors whose unobservable creditworthiness is Gaussian distributed. If an obligor cannot pay interest or repay the loan at maturity, the loan will be in default. Each bank makes default predictions based on credit scores and deduces probabilities of default. The market has the following structure:

$$\begin{pmatrix} S_1 \\ S_2 \\ \vdots \\ S_n \\ Y^* \end{pmatrix} = \begin{pmatrix} \mathbf{S} \\ Y^* \end{pmatrix} \backsim N \left(\begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & \rho_{12} & \cdots & \rho_{1n} & \rho_1 \\ \rho_{12} & 1 & \cdots & \rho_{2n} & \rho_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \rho_{1n} & \rho_{2n} & \cdots & 1 & \rho_n \\ \rho_1 & \rho_2 & \cdots & \rho_n & 1 \end{pmatrix} \right),$$

where S_i is the credit score of competitor i, $\mathbf{S}$ represents the $[n \times 1]$ column vector of credit scores, and Y^* is the non-observable creditworthiness. The default indicator Y is defined as

$$Y = \begin{cases} 0 & : \quad Y^* \leq c, \\ 1 & : \quad Y^* > c, \end{cases}$$

where c is a threshold value, calibrated form the unconditional probability of default $p := \mathbb{P}\{Y = 1\}$ and $q := \mathbb{P}\{Y = 0\}$, respectively.

A. Conditional distributions

The conditional distribution $\mathbf{S}|Y^*$ can be written following Hamilton (1994), pages 100 – 102, as

$$\begin{pmatrix} S_1 \\ S_2 \\ \vdots \\ S_n \end{pmatrix} |Y^* \backsim N \left(\begin{pmatrix} \rho_1 Y^* \\ \rho_2 Y^* \\ \vdots \\ \rho_n Y^* \end{pmatrix}, \begin{pmatrix} 1-\rho_1^2 & \rho_{12}-\rho_1\rho_2 & \cdots & \rho_{1n}-\rho_1\rho_n \\ \rho_{12}-\rho_1\rho_2 & 1-\rho_2^2 & \cdots & \rho_{2n}-\rho_2\rho_n \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{1n}-\rho_1\rho_n & \rho_{2n}-\rho_2\rho_n & \cdots & 1-\rho_n^2 \end{pmatrix} \right),$$

The conditional distribution, $Y^*|\mathbf{S}$, is Gaussian distributed as

$$Y^*|S_1, S_2, ..., S_n \quad \backsim \quad N\left(\rho_1 S_1 + \frac{\rho_2 - \rho_{12}\rho_1}{1-\rho_{12}^2}\left(S_2 - \rho_{12}S_1\right) + ..., \; 1 - \rho_1^2 - \frac{\left(\rho_2 - \rho_1\rho_{12}\right)^2}{1-\rho_{12}^2} - ...\right)$$

Then,

$$\begin{aligned}
\mathbb{P}\{Y=1|S_i\} &= \int_c^\infty f_{(Y^*|S_i)}(y^*)dy^* \\
&= \Phi\left(-\frac{c-\rho_i S_i}{\sqrt{1-\rho_i^2}}\right), && \text{(A.1)} \\
\mathbb{P}\{Y=1|S_i,S_j\} &= \int_c^\infty f_{(Y^*|S_i,S_j)}(y^*)dy^* \\
&= \Phi\left(-\frac{c-\rho_1 S_1-\frac{\rho_2-\rho_{12}\rho_1}{1-\rho_{12}^2}(S_2-\rho_{12}S_1)}{\sqrt{1-\rho_1^2-\frac{(\rho_2-\rho_1\rho_{12})^2}{1-\rho_{12}^2}}}\right), && \text{(A.2)}
\end{aligned}$$

where $f_{(Y^*|S_i)}$ and $f_{(Y^*|S_i,S_j)}$ denote the density functions of the corresponding random variables, and Φ is the standard normal distribution function.

By taking the first derivative

$$\begin{aligned}
\mathbb{P}\{\mathbf{S}\le\mathbf{s}|Y\} &= (1-Y)\frac{\mathbb{P}\{\mathbf{S}\le\mathbf{s},Y^*\le c\}}{\mathbb{P}\{Y^*\le c\}} \\
&\quad +Y\frac{\mathbb{P}\{\mathbf{S}\le\mathbf{s},Y^*>c\}}{\mathbb{P}\{Y^*>c\}},
\end{aligned}$$

we obtain

$$\begin{aligned}
f_{(\mathbf{S}|Y)}(\mathbf{s}) &= \frac{1-Y}{q}f_{\mathbf{S}}(\mathbf{s})\int_{-\infty}^c f_{(Y^*|\mathbf{S})}(y^*)dy^* \\
&\quad +\frac{Y}{p}f_{\mathbf{S}}(\mathbf{s})\int_c^\infty f_{(Y^*|\mathbf{S})}(y^*)dy^*.
\end{aligned}$$

By the same reasoning, we can derive the density functions for the conditional univariate random variables $S_i|Y^*$ and $S_i|Y$ as

$$f_{(S_i|Y^*)}(s) = \frac{1}{\sqrt{1-\rho_i^2}}\phi\left(\frac{s-\rho_i Y^*}{\sqrt{1-\rho_i^2}}\right),$$

and

$$\begin{aligned}
f_{(S_i|Y)}(s) &= \frac{1-Y}{q}\phi(s)\Phi\left(\frac{c-\rho_i s}{\sqrt{1-\rho_i^2}}\right) \\
&\quad +\frac{Y}{p}\phi(s)\Phi\left(-\frac{c-\rho_i s}{\sqrt{1-\rho_i^2}}\right),
\end{aligned}$$

where ϕ denotes the standard Gaussian density function.

B. ROC curve

On the basis of the density function $f_{(S_i|Y)}$, we calculate α- and β-error, dependent on the threshold value t, namely

$$\begin{aligned}
\alpha_i(t) &= \mathbb{P}\{S_i \le t | Y = 1\} \\
&= \frac{1}{p}\int_{-\infty}^{t} \mathbb{P}\{Y = 1 | S_i = s\}\,\phi(s)ds \qquad \text{(A.3)} \\
&= \int_{-\infty}^{t} f_{(S_i|Y=1)}(s)ds \\
&= \frac{1}{p}\int_{-\infty}^{t} \phi(s)\Phi\left(-\frac{c-\rho_i s}{\sqrt{1-\rho_i^2}}\right)ds \qquad \text{(A.4)}
\end{aligned}$$

The first line reflects the definition of the α-eroror, under the null hypothesis that obligors with a credit score higher than t are defaulters. With the second type of error, we can proceed the same way

$$\begin{aligned}
\beta_i(t) &= \mathbb{P}\{S_i > t | Y = 0\} \\
&= \frac{1}{q}\int_{t}^{\infty} \mathbb{P}\{Y = 0 | S_i = s\}\,\phi(s)ds \qquad \text{(A.5)} \\
&= \int_{t}^{\infty} f_{(S_i|Y=0)}(s)ds \\
&= \frac{1}{q}\int_{t}^{\infty} \phi(s)\Phi\left(\frac{c-\rho_i s}{\sqrt{1-\rho_i^2}}\right)ds \qquad \text{(A.6)}
\end{aligned}$$

For a given threshold level t, $(\beta(t), 1-\alpha(t))$ constitutes one coordinate of the ROC curve. The curve is drawn by running t from $-\infty$ to $+\infty$. AUROC denotes the area below the ROC curve and is computed by

$$\begin{aligned}
\text{AUROC}_i &= -\int_{-\infty}^{\infty} (1-\alpha_i(t))\,d\beta_i(t) \\
&= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathbf{1}\{x > t\}\, f_{(S_i|Y=0)}(x) f_{(S_i|Y=1)}(t)dxdt \qquad \text{(A.7)} \\
&= \frac{1}{pq}\int_{-\infty}^{\infty} \phi(t)\Phi\left(\frac{c-\rho_i t}{\sqrt{1-\rho_i^2}}\right)\int_{t}^{\infty} \phi(x)\Phi\left(-\frac{c-\rho_i x}{\sqrt{1-\rho_i^2}}\right)dxdt
\end{aligned}$$

The first two lines are true for all continuous credit scores, and the last line follows by the normal assumption. From (A.7) AUROC can be nicely interpreted as the probability that the continuous credit score of a non-defaulting obligor is lower than the one of a defaulting obligor, given the two scores were drawn independently. The slope of the ROC curve, $s(t) := -\frac{d\alpha(\beta(t))}{d\beta(t)}$, can be represented as

$$s_i(t) = -\frac{\frac{d\alpha_i(t)}{dt}}{\frac{d\beta_i(t)}{dt}} = \frac{q}{p}\frac{\Phi\left(-\frac{c-\rho_i t}{\sqrt{1-\rho_i^2}}\right)}{\Phi\left(\frac{c-\rho_i t}{\sqrt{1-\rho_i^2}}\right)} \tag{A.8}$$

where the second equality follows from (A.4) and (A.6).

Figures

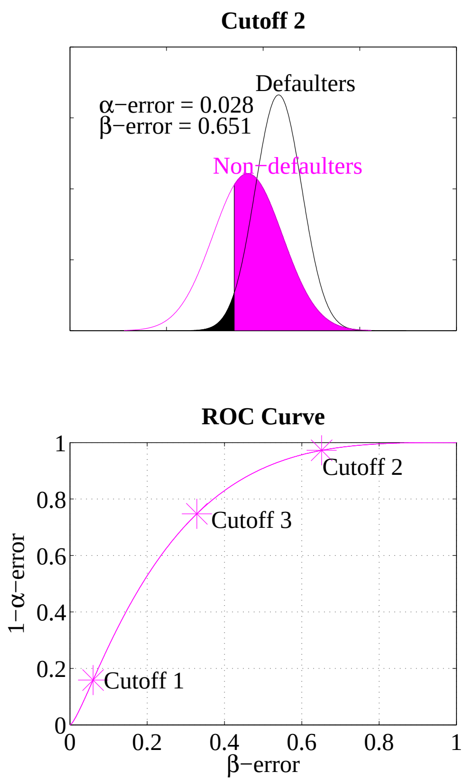

Figure 1. Construction of a ROC curve. The types of error refer to the null hypothesis that the higher the credit score, the higher the default risk.

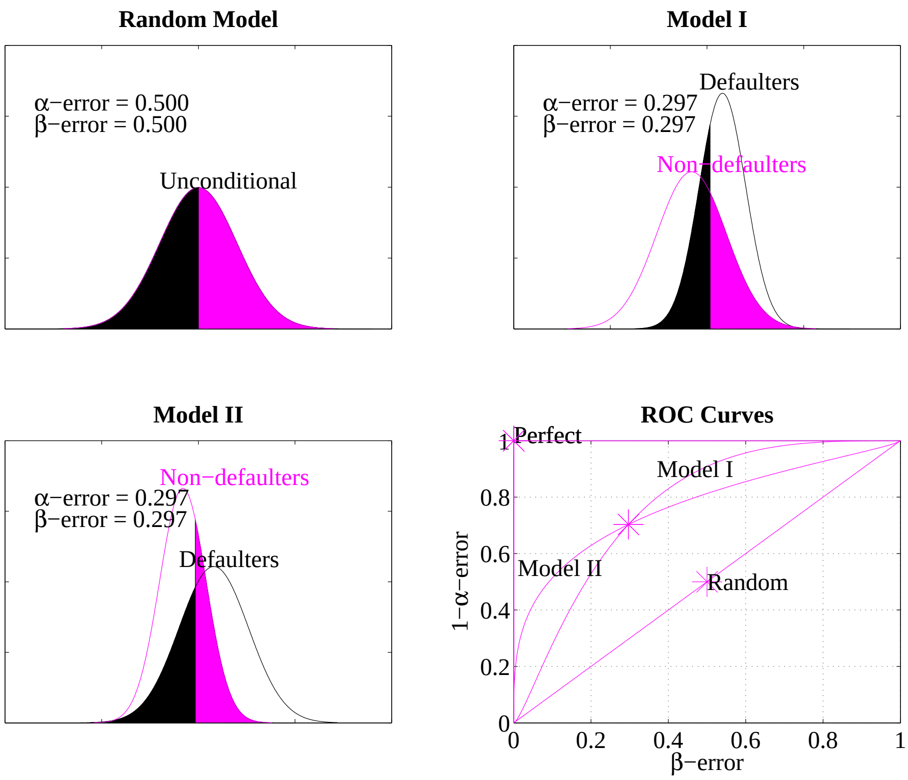

Figure 2. Comparison of ROC curves.

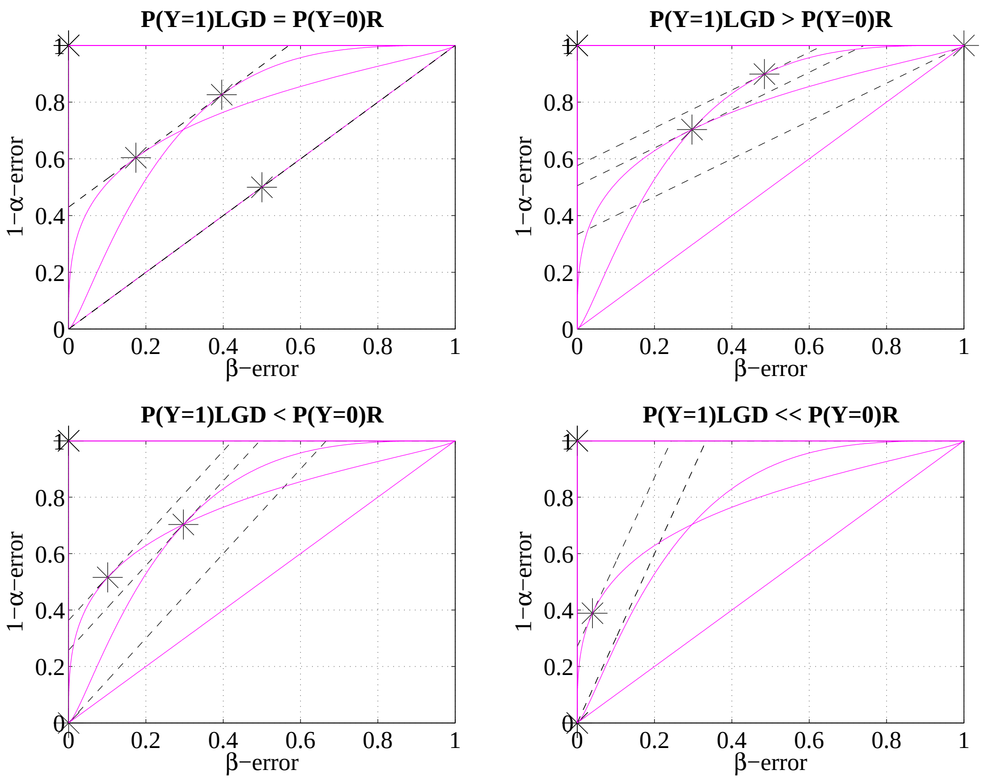

Figure 3. Trade-off between $\alpha-$ and $\beta-$error under different cost (LGD) and revenue (R) assumptions.

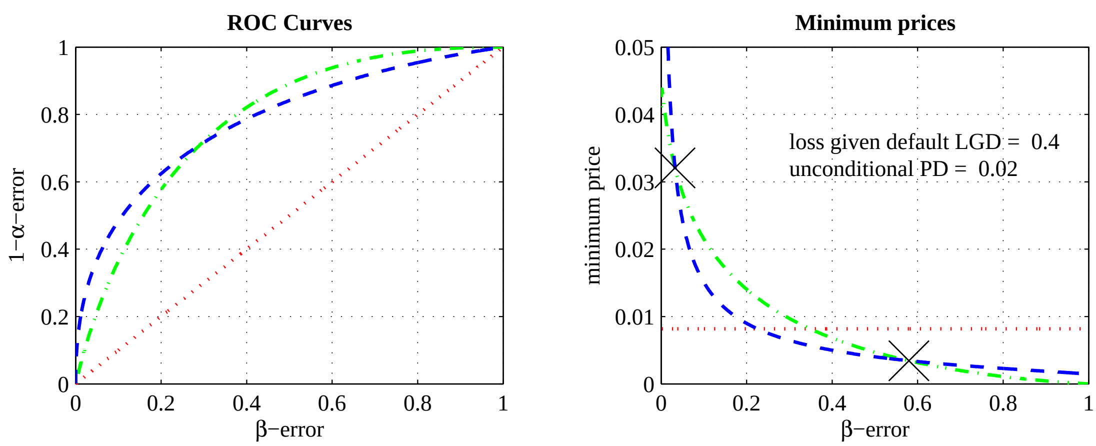

Figure 4. Same area below ROC curve may lead to different pricing schemes. By applying a random model no price discrimination is possible, whereas a perfect model simply refuses all defaulting obligors.

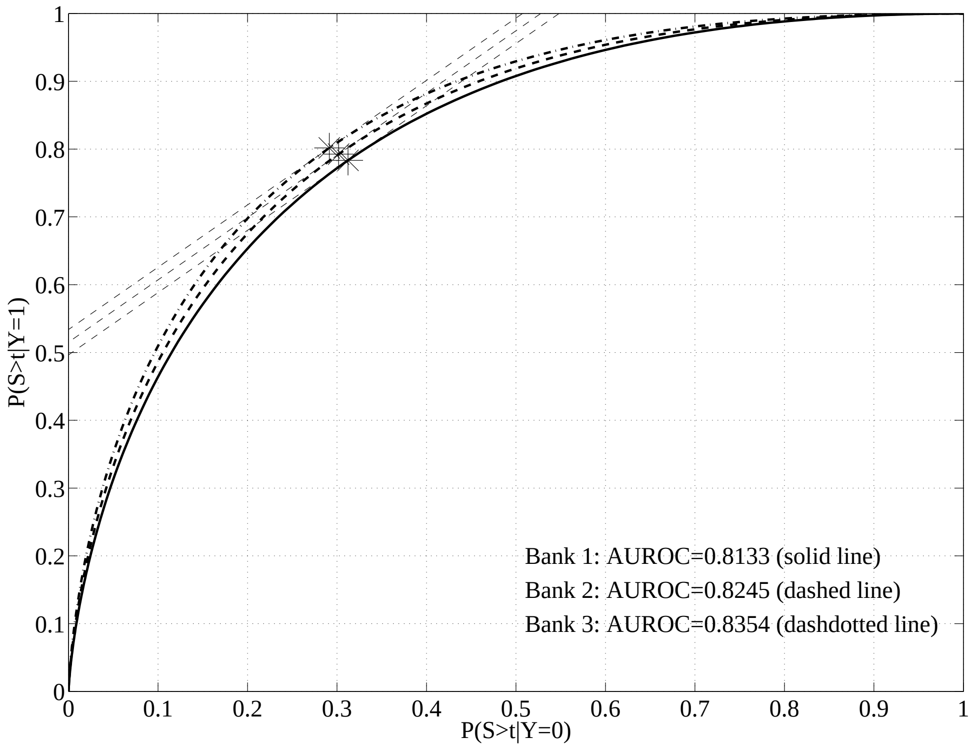

Figure 5. ROC curves of the three banks with model-specific optimal cutoffs and corresponding error rates, for given cash-flow assumptions. Bank 1 (Bank 2, Bank 3) accepts 67.8% (68.8%, 69.8%) of all obligors according to profit-optimal cutoffs of 0.4628, 0.4912 and 0.5199.

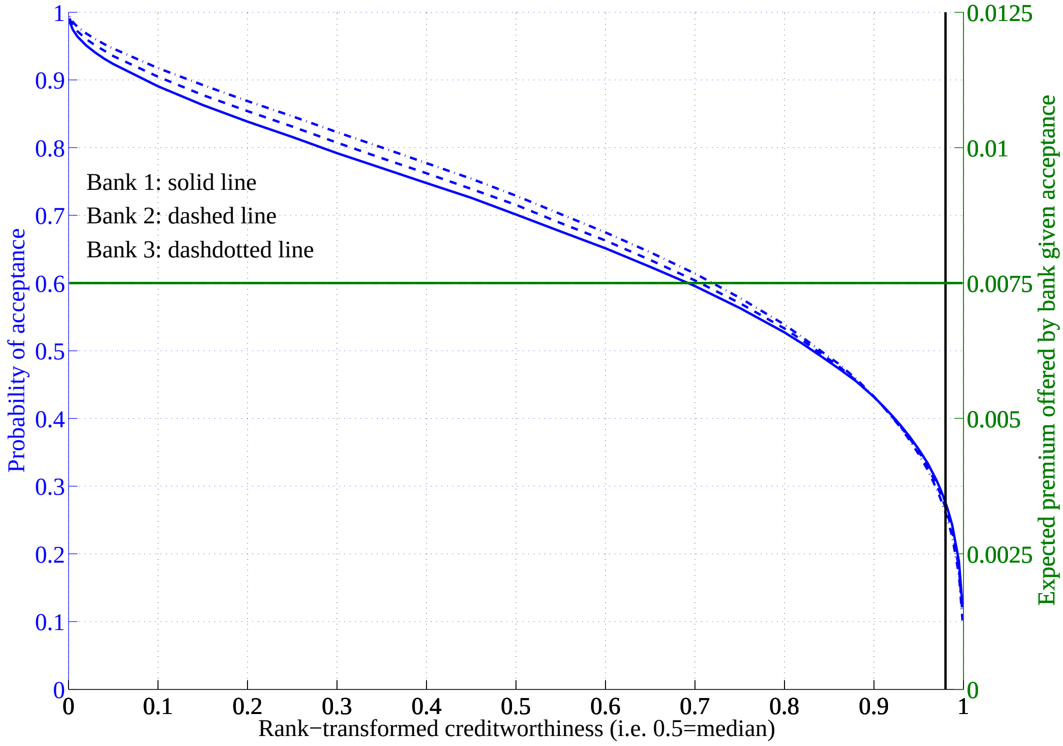

Figure 6. Cutoff regime: Probability of acceptance versus creditworthiness with y-axis labeling on the left and expected offered premium versus creditworthiness with y-axis labeling on the right. Credit spread is fixed.

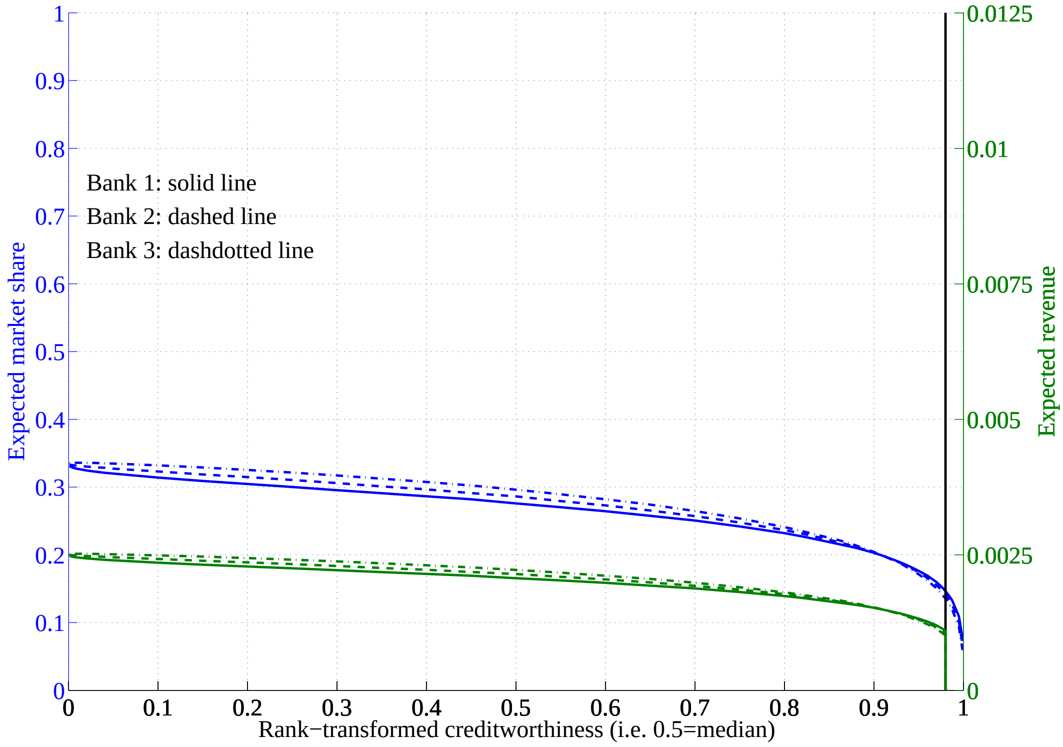

Figure 7. Cutoff regime: Expected market share versus creditworthiness with y-axis labeling on the left and expected revenue versus creditworthiness on the right.

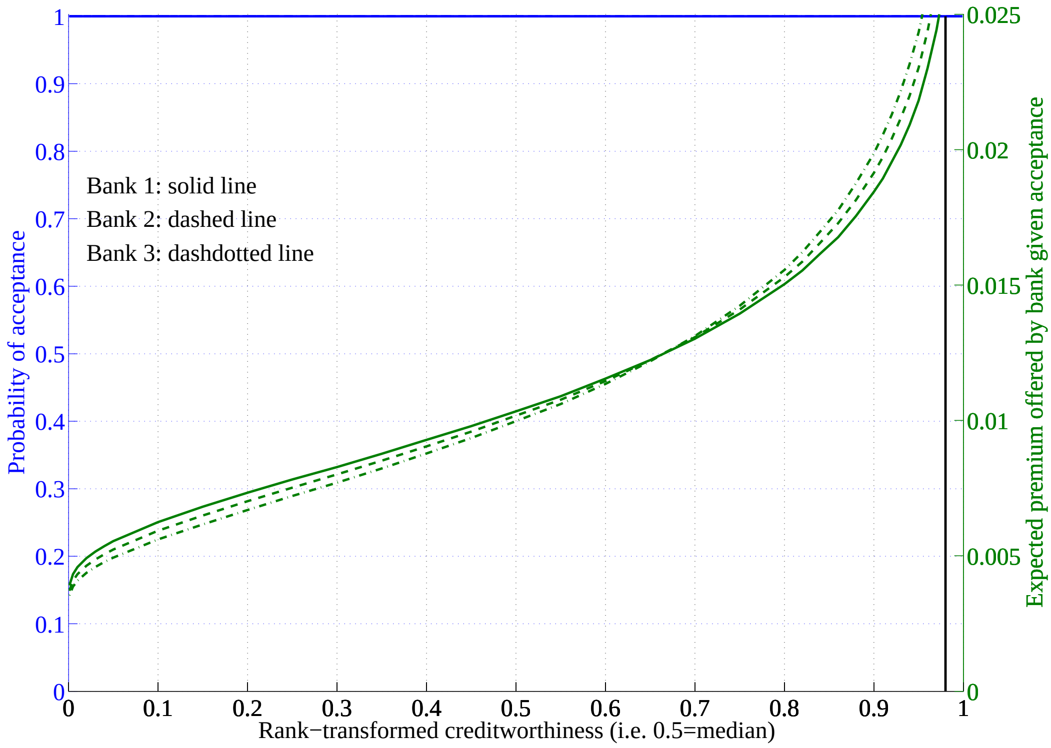

Figure 8. Pricing regime: Probability of acceptance versus creditworthiness with y-axis labeling on the left and expected offered premium versus creditworthiness on the right. Probability of acceptance is 1.

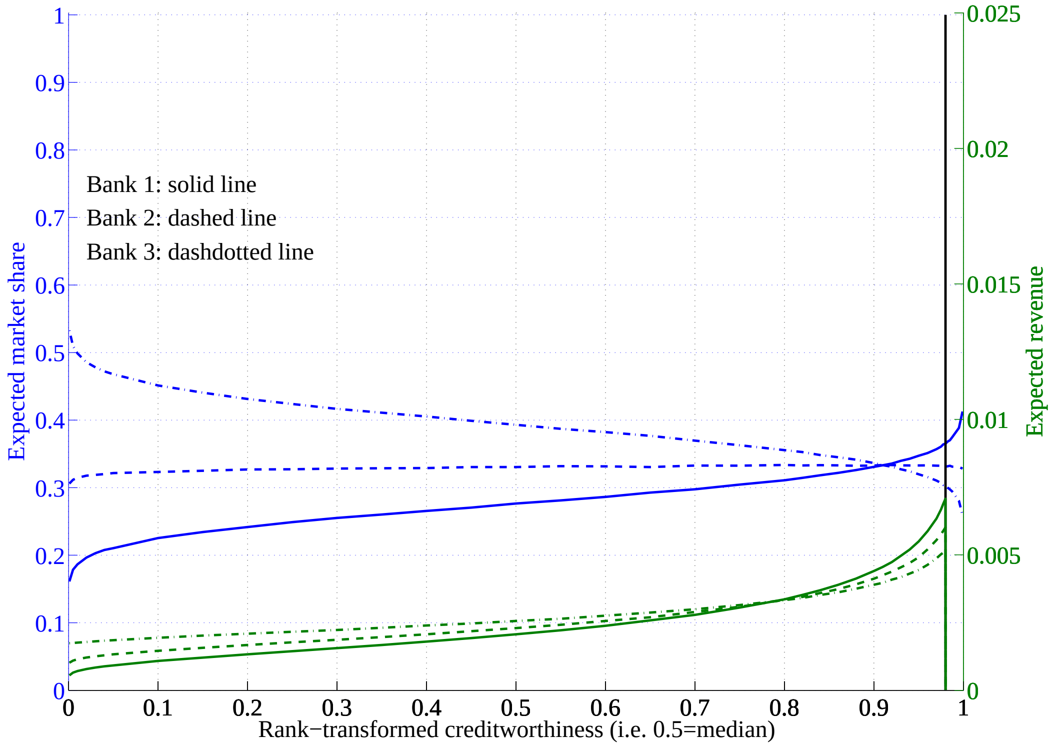

Figure 9. Pricing regime: Expected market share versus creditworthiness with y-axis labeling on the left and expected revenue versus creditworthiness on the right.

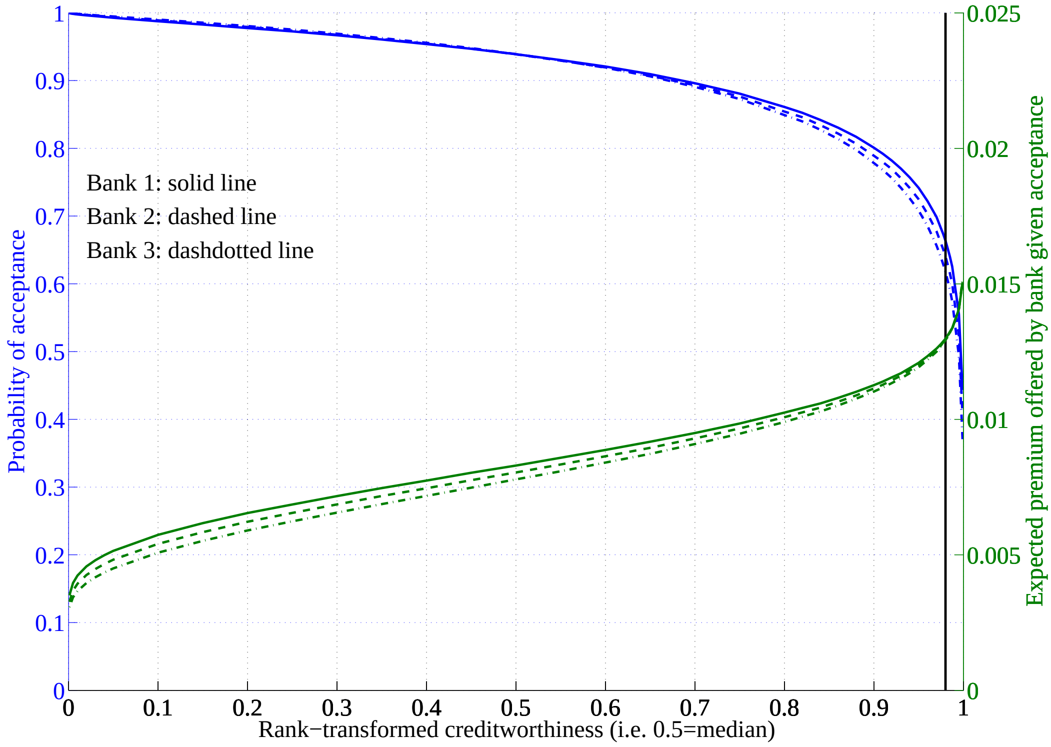

Figure 10. Mixture regime: Probability of acceptance versus creditworthiness with y-axis labeling on the left and expected offered premium versus creditworthiness on the right.

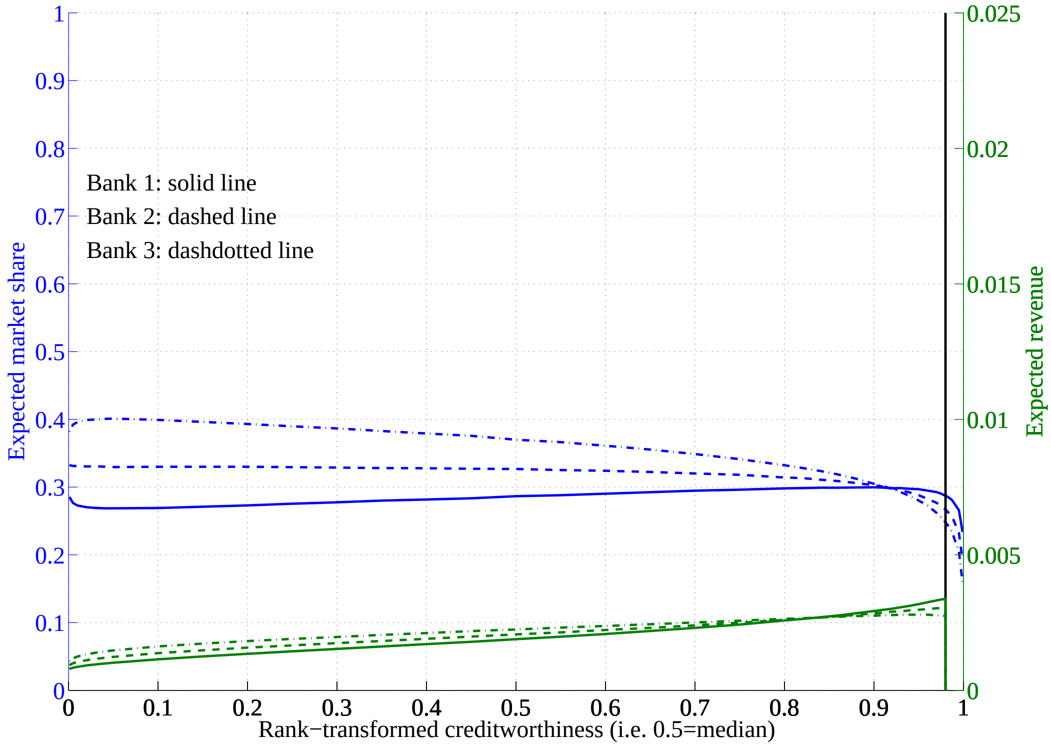

Figure 11. Mixture regime: Expected market share versus creditworthiness with y-axis labeling on the left and expected revenue versus creditworthiness on the right.

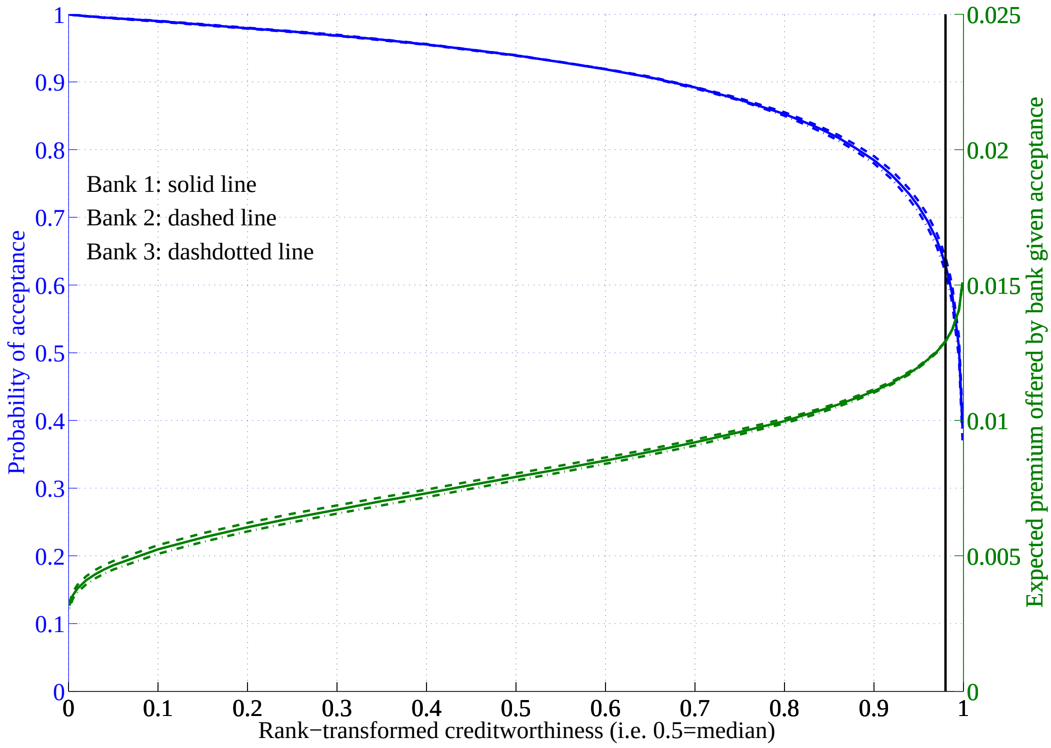

Figure 12. Mixture regime – Bank 1 improves: Probability of acceptance versus creditworthiness with y-axis labeling on the left and expected offered premium versus creditworthiness on the right.

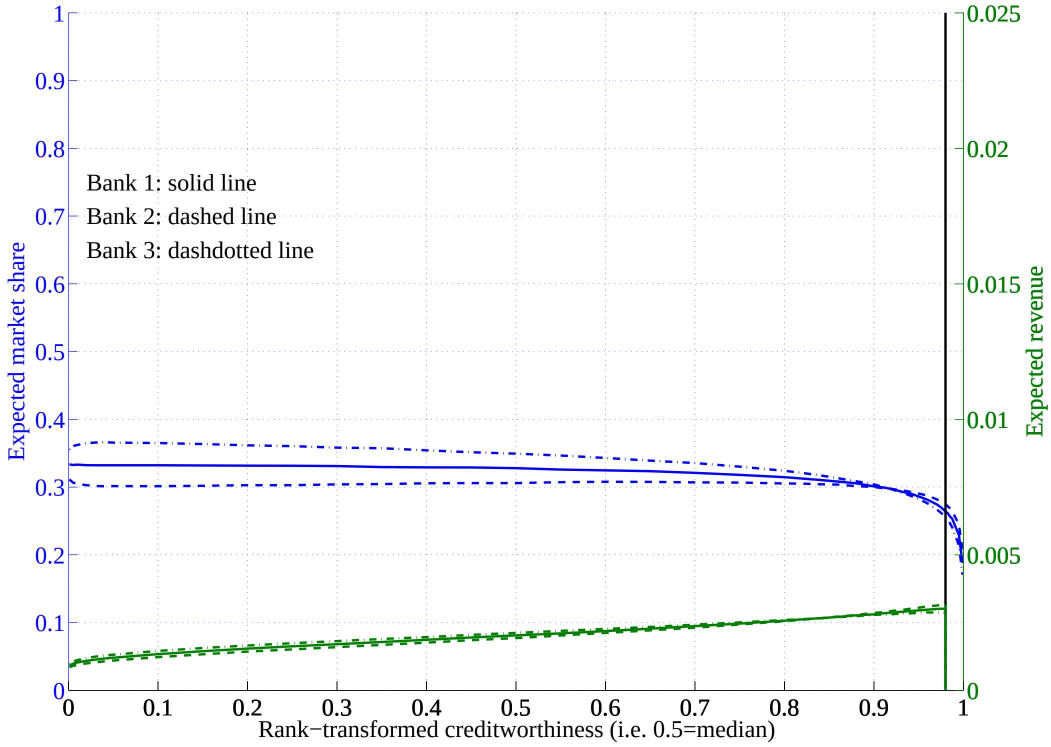

Figure 13. Mixture regime – Bank 1 improves: Expected market share versus creditworthiness with y-axis labeling on the left and expected revenue versus creditworthiness on the right.

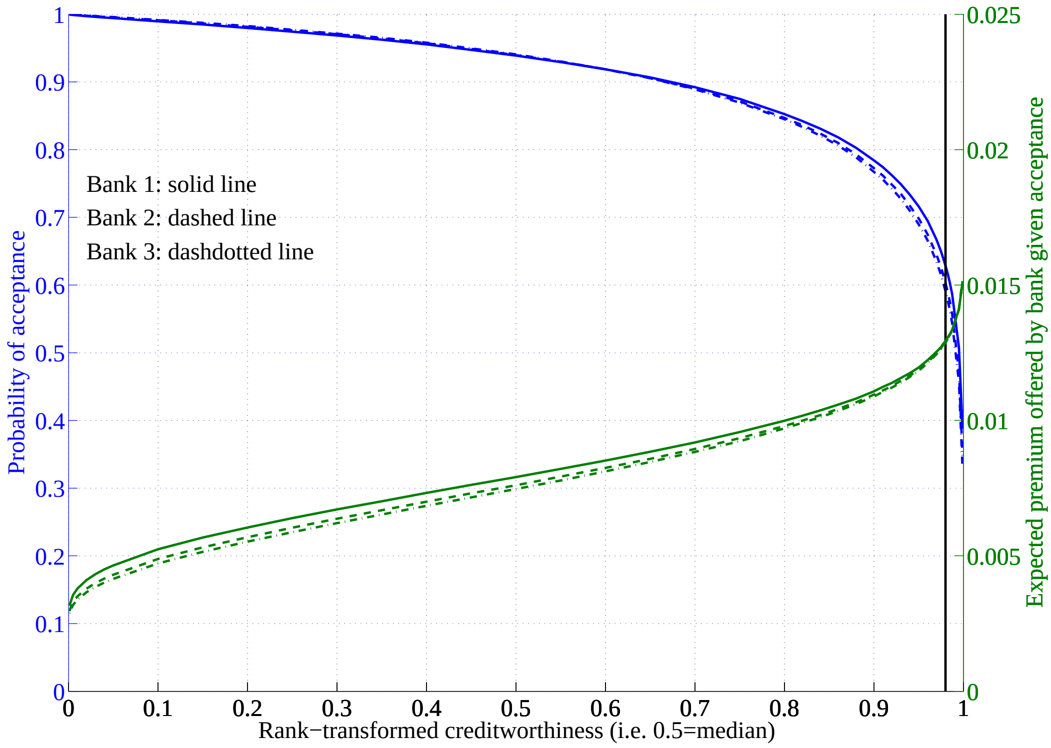

Figure 14. Mixture regime – Bank 1 improves, but Bank 2 and Bank 3 know methodology: Probability of acceptance versus creditworthiness on the left and expected offered premium versus creditworthiness on the right.

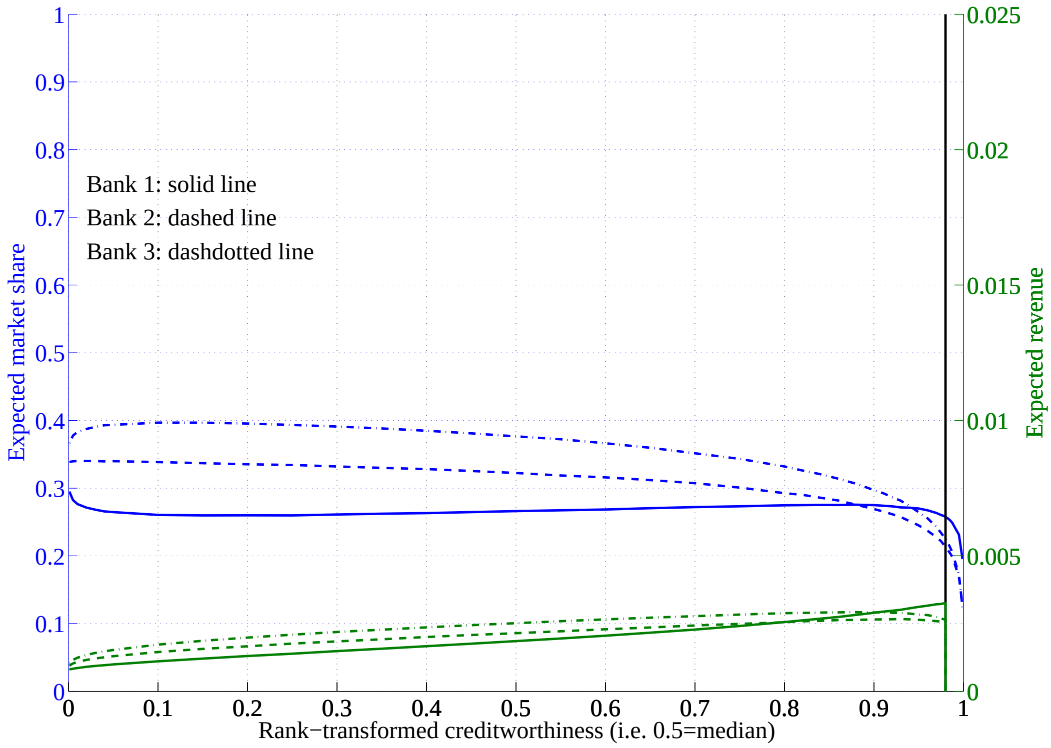

Figure 15. Mixture regime – Bank 1 improves, but Bank 2 and Bank 3 know methodology: Expected market share versus creditworthiness on the left and expected revenue versus creditworthiness on the right.

Tables

		Actual	
		Low Credit Quality	High Credit Quality
Model	Low Credit Quality	Correct assessment	Lost potential profits, and opportunity costs. Lost interest income and fees. Divest credits at disadvantageous conditions.
	High Credit Quality	Lost interest and credit amount through defaults. Recovery costs.	Correct assessment

Table 1

Costs of errors

	Default	**Non-Default**
Cash flow	Loss given default, recovery cost LGD	Interest and fees + relationship benefit $R(t) + C$
Probability	$\mathbb{P}\{Y = 1 \vert S = t\}$	$\mathbb{P}\{Y = 0 \vert S = t\}$

Table 2

Cash flows and probabilities conditional on score $S = t$

market figure	Bank 1	Bank 2	Bank 3	Total
AUROC of model	0.8134	0.8245	0.8354	—
market share	0.2658	0.2729	0.2803	0.8189
share non-defaulters	0.2687	0.2762	0.2838	0.8287
share defaulters	0.1208	0.1135	0.1064	0.3407
profit	100.9	112.1	123.5	336.5
revenue	197.5	203.0	208.6	609.1
loss	96.6	90.8	85.1	272.6

Table 3

Market shares as well as profit, revenue & loss in million USD in the cutoff regime

market figure	Bank 1	Bank 2	Bank 3	Total
AUROC of model	0.8134	0.8245	0.8354	—
market share	0.2771	0.3292	0.3937	1.0000
share non-defaulters	0.2749	0.3291	0.3959	1.0000
share defaulters	0.3821	0.3308	0.2870	1.0000
profit	-74.6	-19.0	36.0	-57.7
revenue	231.0	245.5	265.5	742.0
loss	305.6	264.6	229.5	799.7

Table 4

Market shares as well as profit, revenue & loss in million USD in the pricing regime

market figure	Bank 1	Bank 2	Bank 3	Total
AUROC of model	0.8134	0.8245	0.8354	—
market share	0.2846	0.3200	0.3597	0.9643
share non-defaulters	0.2848	0.3216	0.3626	0.9691
share defaulters	0.2709	0.2426	0.2171	0.7306
profit	-27.3	7.6	41.8	22.1
revenue	189.3	201.6	215.4	606.4
loss	216.6	194.0	173.6	584.3

Table 5

Market shares as well as profit, revenue & loss in million USD in the mixture regime

market figure	Bank 1	Bank 2	Bank 3	Total
AUROC of model	0.8300	0.8245	0.8354	—
market share	0.3208 (+0.0362)	0.3026 (-0.0174)	0.3401 (-0.0196)	0.9635 (-0.0008)
share non-defaulters	0.3225 (+0.0376)	0.3036 (-0.0180)	0.3424 (-0.0202)	0.9685 (-0.0005)
share defaulters	0.2393 (-0.0316)	0.2526 (+0.0106)	0.2258 (+0.0087)	0.7177 (-0.0128)
profit	7.0 (+34.3)	-10.0 (-17.6)	24.4 (-17.4)	21.4 (-0.7)
revenue	198.4 (+9.1)	192.1 (-9.5)	205.1 (-10.3)	595.6 (-10.7)
loss	191.4 (-25.2)	202.1 (+8.1)	180.7 (+7.1)	574.2 (-10.0)

Table 6

Mixture regime: Bank 1 improves AUROC from 0.8134 to 0.8300; market shares as well as profit, revenue & loss in million USD; in brackets, the difference to case of Bank 1's AUROC of 0.8134

market figure	Bank 1	Bank 2	Bank 3	Total
AUROC of model	0.8300	0.8245	0.8354	—
market share	0.2663 (-0.0545)	0.3115 (+0.0089)	0.3593 (+0.0192)	0.9372 (-0.0264)
share non-defaulters	0.2669 (-0.0556)	0.3141 (+0.0105)	0.3628 (+0.0204)	0.9439 (-0.0247)
share defaulters	0.2378 (-0.0015)	0.1830 (-0.0696)	0.1889 (-0.0369)	0.6097 (-0.1081)
profit	-4.3 (-11.3)	57.1 (+67.0)	83.3 (+58.9)	136.0 (+114.6)
revenue	186.1 (-12.3)	203.7 (+11.5)	234.7 (+29.6)	624.5 (+28.8)
loss	190.5 (-0.9)	146.6 (-55.5)	151.4 (-29.3)	488.5 (-85.8)

Table 7

Mixture regime: Bank 2 and Bank 3 know rating methodology of Bank 1; market shares as well as profit, revenue & loss in million USD; in brackets, the difference to case of proprietary rating methodologies.

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